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CONTENTS:

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*THE GROUP-VELOCITY AND THE WAVE-VELOCITY OF LIGHT.**

ALTHOUGH the determination of the important constant of nature—the velocity of light—has occupied the attention of scientists from the time of Galileo, and while astronomical and terrestrial methods have been so carefully refined that individual observers have obtained values differing by less than one part in 3,000, it is a significant fact that no terrestrial method thus far used gives the absolute velocity of light under all conditions. If a group of periodic disturbances are radiated out into any medium the velocity of the individual elements will in general be different from that of the mean of the group. Only in the one instance, the propagation in vacuo, is it likely that these two velocities are the same; and here physical methods, thus far, have not put the question to a test. In the case of ponderable media important data are to be expected. The astronomical method used by Römer in 1675 and founded on the observation of the eclipse of Jupiter's satellites gives the so-called group-velocity of light in vacuo. The observation of the fixed stars discovered by Bradley in 1727 gives the wave-

* Address of the retiring President and Chairman of Section B—Physics—of the American Association for the Advancement of Science, Pittsburgh meeting, June 28 to July 3, 1902.

velocity in the medium within the observing telescope. This is the only method thus far used which gives the absolute velocity of light.

The uncertainty in the constant of aberration and the errors in the observations of Jupiter's satellites render these methods unsuited for the comparison of the two velocities. We owe perhaps to Arago more than any other person a solution of this problem of the velocity of light. In 1838 he submitted to the French Academy the plan of an experiment for solving directly and definitely the question—which was being much debated—whether light was a corpuscular emission from radiant bodies or simply the result of the vibrations of a very rare medium. The method was simply an adaptation of the rotating mirror ingeniously devised by Wheatstone for determining the velocity of propagation of electricity in a wire. It was not till 1850, however, that this method, in the hands of Foucault, was actually put to the test of determining this constant. In the preceding year, however, Fizeau had published the results of his experiments by means of a toothed wheel. These were the first observations made on the velocity of light at the earth's surface. The first idea of this method seems to have originated with the Abbé Laborde, who communicated it to Arago some years before. Upon these two principles is based our entire knowledge by terrestrial methods of this constant. On the one side we have the refinement and modifications of the toothed wheel as used by Cornu in France and Young and Forbes in England and on the other the very accurate results of Michelson in his classic experiments and those of Newcomb with his highly refined phototachometer in this country. It may well be questioned whether much greater certainty in data is attainable than those which the late lamented French savant and illustrious physicist Cornu has left as one of his most brilliant legacies to science. Nor can we hope for any considerable refinement in the determinations by the other method as used in this country, and which have already given the most surprising agreement and warrants us in taking the value obtained as the most accurate of all. Notwithstanding the elaborate execution of these experiments serious discrepancies exist between the results of the two methods. The latest discussion by Cornu of his own results gives as the most probable value of this constant 300,130 meters, while the values of Michelson and of Newcomb are 299,853 and 299,810, respectively. This makes the error of the results by the two methods ten times the error between those of the same method. This difference has given rise to the impression that in the one or the other or in both methods a systematic error exists and the discussions and corrections made by different critics have left the problem in a somewhat uncertain state. The relation of these determinations to the absolute velocity in air at the earth, and to the absolute terrestrial velocity in vacuo and the possible difference from the velocity in space, renders the problem of great interest and importance in this aspect alone. The famous theory of Weber of moving charges to explain the action of electric currents, while incompatible with the principle of the conservation of energy, has done much to enlarge our views of the origin of electrodynamic phenomena and to establish a comprehensive theory of present phenomena. The brilliant experiment of Rowland as a natural sequence of Weber's theory demonstrated the electromagnetic reaction of a moving charge and showed further that if the velocity were that of light the mechanical reaction would be approximately that calculated from

theory by using the value of the light constant as the ratio of the two units.

The prediction by Maxwell that light was an electromagnetic disturbance in the medium surrounding an oscillating charge, and the consequent identity of the velocity of light in the ether alone with the ratio of the electrical units in the two systems of measurements used, when a charge is respectively in motion or at rest and the further relation of the light constant to the dielectric constant for ponderable media, have been since fully confirmed by exhaustive experiments. His interpretation of the physical significance of the ratio of the electromagnetic unit to the electrostatic unit as a velocity of the same magnitude as that for light received remarkable confirmation in the independently conceived experiment of Rowland already referred to.

The celebrated experiments of Hertz on electric oscillations and the identification of the velocity of their propagation in the ether with that of light waves constitute perhaps a more remarkable instance of the confirmation of a brilliant conception than that of the law of gravitation itself.

If we accept these facts as confirming the supposition that light is an electric phenomenon, then we may consider the results found as data obtained by different methods for the solution of the problem, the velocity of light. It would be necessary then to examine the principles of the methods involved to determine what phase of the problem each corresponds to, *i. e.*, whether to a group-velocity or to a wave-velocity.

Consider first v, the ratio of the two units. In the derivation of the equations for the propagation of undulations in a non-conducting medium the time rate of change in the polarization, either electric or magnetic, is obtained in terms of the line integral of the force, magnetic or electric respectively, around the bounding curve through which this polarization or flux takes place. Since now each term in the resulting equations may be expressed in either the electrostatic or electromagnetic units, the integral of these differential equations would show some connection between the constant in the problem and the ratio of the units, if different units are used, otherwise not. The well-known solution of these so-called wave-equations is a wave-potential involving as one of its factors a function periodic in time and in space. If we follow any value of the function, *i. e.*, the same phase of the disturbance, the distance we shall have gone in a unit of time is found to be the number of electrostatic units in the electromagnetic unit multiplied into the reciprocal of the square root of the constants of electric and magnetic polarization, respectively. In vacuo these constants are unity. We therefore conclude that the value of v is the wave-velocity of light and not the group-velocity.

In the experiment for measuring the velocity of propagation of electric oscillations or Hertzian waves, the frequency of these oscillations is determined either directly, by observing the successive discharges in a rotating mirror, or by calculations from the capacity and induction of the electrical system. By determining the wave-length of disturbance—usually by noting the nodes of standing waves along a wire—the velocity is found. The velocity may also be measured by noting the time for the transmission of individual disturbances over a given interval of space. These methods all have to do with a phase of the disturbance and not with the mean of a group of oscillations, and hence correspond to the wave-velocity.

The electrical methods then all give the wave-velocity, while the optical methods

thus far used do not give directly the wave-velocity or the velocity of the individual disturbance, but a velocity dependent on that of the group.

While the agreement between these electrical constants and the light constant has perhaps been the strongest factor in the identification of electromagnetic and optical phenomena, additional discoveries now give incontrovertible evidence of the common agency of the two classes of phenomena, so that these constants may now be considered with good reason to be, not so valuable as evidence of like phenomena, as independent data in determining the true value of the velocity of propagation of the medium for electrical and optical disturbances. It is true that exact quantitative evidence is lacking. The experiment of Rowland is essentially qualitative, and although his results agree approximately with values calculated from theory, more exact results are extremely desirable, although such a possibility seems to transcend present mechanical attainments. The futile attempts to definitely establish by direct experiments the electrodynamic relations between electric charges and the electromagnetic field do not disturb our confidence in the truth of the theory.

Experiment still fails to give us a mechanical reaction on a charged particle moving in a magnetic field. It fails also in giving a positive reaction on a charged particle when the magnetic field is varied. The experiments to detect the electromotive intensity produced by the variation of the velocity of a moving charge have not yet been successful. These are all essential features of the electromagnetic theory and undoubtedly will receive a successful solution in the future. On the other hand the action of a magnetic field in affecting the discharge in a vacuum electrode tube and the celebrated discovery by Hall designated as the Hall 'effect,' are evidence of the reality of the mechanical action on a charge moving in a magnetic field. The phenomena of discharge, in electrodeless tubes in the presence of electric oscillations is significant of the mechanical action on a charged particle in a varying magnetic field. The discoveries by Faraday and by Zeemann—as we now interpret the association of electrical charges on matter as evidenced by what we know from electrolysis—are a further confirmation of the mechanical reaction of a field of force upon moving charges. The experiments of Lecher on the magnetic action of displacement currents in a dielectric also confirm our ideas in regard to the essential characteristic of an insulating medium and the electric charges on the ultimate elements of matter. Hence we are with full reason bound to identify these constants, and may therefore examine their derivation by a closer analysis of their real significance. If on the optical side the problem of the velocity of propagation of individual disturbances has never been attacked directly, there seems to be full reason for doing so in order to complete the evidence from the standpoint of light phenomenon which we already have at hand on the electrical side.

It would be desirable to determine the velocity of a group of periodic electric disturbances under varying conditions in order to compare it with the velocity of a single disturbance.

In the methods of Fizeau and of Foucault, which are the only ones used thus far, the time of the 'go' and return of a flash of light is measured. The relation of this time to that of the time of the 'go' and return of a single one of the component waves is not a relation at once simple and evident. No experiments have been directly carried out to determine this relation in optical media. We have the-

oretical considerations of analogous examples to go by, but no direct experimental data. Lord Rayleigh has considered the problem. It has been noticed that in the progress of a group of waves in water, the individual waves appear to advance through the group and die away at the anterior limit. Stokes has explained this by regarding the group as formed by the superposition of two infinite trains of waves of equal amplitudes and of nearly equal wave-lengths advancing in the same direction. The mathematical formulation of this phenomenon as thus explained gives a resultant periodic motion with a periodic amplitude varying from zero to the sum of the two elements. The velocity of this maximum, which is called the group-velocity U is related to the wave-velocity V by the variation with respect to the wave-length λ. If the wave-velocity V is definitely known as a function of the wave-length, then the group-velocity can be ascertained. On the other hand, we cannot determine the wave-velocity V from a complete knowledge of the function U. It is necessary that we know the relation of V to the wave-length. Rayleigh finds that $U=(1-n)V$ if the wave-velocity V varies as the nth power of the wave-length λ. Thus for deep-water waves $n=1/2$, $U=3/2V$. In the case of aerial waves U and V are nearly the same. In this instance the ear detects the periodic variation of the resultant amplitude as beats which are propagated out with the velocity of the component waves. The resultant of two such systems of light waves may be illustrated by the interference of the two sodium lines in Newton's rings and the periodic variation in the luminosity of the rings when a great number are examined together. This of course is the fluctuation which occurs in the resultant radiations propagated into space but not capable of being seen by the eye.

The argument from the kinematical point of view for the relation of the two velocities is not entirely beyond criticism as this requires a gradual variation in the amplitude according to the cosine law. As the group sent out by either of these two methods must deviate considerably from this law, it would be necessary to include a number of harmonics in Fourier's series to give the proper configuration to the group. In order that we may then use the kinematical argument we must assume these harmonics are rapidly frittered down and that they never return. This may have some significance in the toothed-wheel method, where some observers have noted a coloration of the return image. Further analysis of the kinematical problem is necessary before we can feel sure of its application to the physical counterpart. The argument which Lord Rayleigh has advanced, based on the consideration of the energy propagated, assumes absorption due to a frictional term proportional to the velocity. Now while absorption in ponderable media is explained on the assumption of imbedded particles in the ether of some specific period, it has not yet been proven that this is the only way in which absorption may take place. If there be absorption in the ether itself it is not easy to see just how it does occur. On the assumption already made above it would be impossible for the ether to transmit waves of certain frequencies corresponding to its natural period and we should have selective absorption, a condition quite contrary to the conception of such a medium. On the above assumption, however, the ratio of the energy passing a given point in a unit of time to the energy in the train after this unit of time is the ratio of the group-velocity to the wave-velocity. Thus we see the ratio depends directly on the amount of absorption. It is not quite clear,

however, that this relation would hold for absorption by some other mode. We may then feel some hesitancy in accepting this relation of the group-velocity to the wave-velocity as based on either the kinematical analogy or the energy argument. We must therefore fall back for the solution upon direct experimental means. The significance of such an experimental solution to the problem of the propagation of undulations should not be underestimated. Investigations of these two velocities for monochromatic light, such as from cadmium or mercury, in highly dispersing substances like carbon disulphide or alpha-monobromonapthaline or dense glass of Faraday, now seem entirely possible and sufficient for the solution of the problem.

In the case of the ether the arguments which can be advanced in regard to velocity of light for different colors indicate the same velocity for all colors. It was pointed out by Arago that any difference in velocity should produce a coloration in any luminous body in the universe which should vary rapidly in intensity. Thus in the observations on the eclipse of Jupiter's satellites they should momentarily show at the instants of disappearing and reappearing a coloration complementary in the two cases. Nothing of this kind has been recorded. Again in the case of Algol, Newcomb estimated, from its probable distance, —greater than 2,000,000 radii of the earth's orbit and the time for light to reach us, 30 years—that a difference in time between the blue and the red rays of one hour would give a difference in velocity of four parts in a million. In the remarkable changes in Nova Persei last year, its complete spectrum appeared to be visible even though the changes in its intensity were far more rapid than in the case just mentioned. As no trace of coloration has ever been observed this difference of time cannot exceed a fraction of an hour. It should be mentioned, however, that in the experiments of Young and Forbes, the velocity of blue light was apparently about 1.8 per cent. greater than that of red light. Both Michelson and Newcomb claim that this would have been very distinctly observed in their experiments with the rotating mirror in the spreading out of the image of the slit into a distinct spectrum. A further instance is cited by Lord Rayleigh which may be of value. If we examine the position of the bands in the spectrum of glowing gases, we find certain harmonic relations. Now if these rays had different velocities in the free ether the position of the bands would be affected and the harmonic relations, apparently holding as deduced from the spectra observed, would not give the harmonic relations in the radiants themselves; or, *vice versa,* such harmonic relations between the radiants would not give harmonic distribution of the bands in the spectrum. From another standpoint it may be mentioned that on any theory of an optical medium we know that either a difference in velocity or a dispersion requires incomplete transmission. This may be due to internal reflection or to transformation into heat. The transmission would also be differential. Thus only a part of the light of the distant stars would reach the eye and this would be more and more colored as the distance increased, due to the differential transmission. No effect of this kind can be observed even in the nebulæ which are so remote that the telescope cannot resolve them, although the spectroscope gives us unquestioned evidence of their stellar nature.

These arguments from the astronomical point of view are, however, uncertain and indirect. Until we can determine to a close approximation the wave-velocities of different colored rays in ponderable media as

well as in the ether, we cannot be assured that we are entitled to consider the determination by the group methods used heretofore as sufficient to give us the absolute velocity of light. Even if we regard the evidence from astronomical observations of the common velocity in space for all colors, and from this conclude that the absolute velocity is the group-velocity, as the equations of Lord Rayleigh show with the assumptions he makes, we are still lacking sufficient data for the relation in the case of ponderable media.

In the discussion of the results by the toothed-wheel method and the rotating-mirror method, considerable difference of opinion has been expressed as to just what we obtain by the latter method. There seems to be no dissenting opinion that the toothed-wheel method gives the group-velocity directly, for here we have the time of transit of an interrupted beam of light. In the rotating-mirror method the ray is also intermittent. Lord Rayleigh seems first to have raised the interesting question as to what is actually measured in these experiments, and in his first note states that the rotating-mirror method gives the group-velocity U. In a later article he arrives at a different result and gives the square of the wave-velocity divided by the group-velocity, V^2/U. Evidently unless we know the relation of the two we can find neither if this be correct. Now this relation is not certainly known as pointed out above. On the other hand Gouy, however, dissents from this second view and shows that the group-velocity U is the quantity determined by the rotating-mirror method. Schuster in a later article dissents from Gouy's conclusion and corrects Lord Rayleigh's second result, and gives the square of the wave-velocity divided by twice the wave-velocity minus the group velocity, $V^2/(2V-V)$. J. Willard Gibbs in a later article points out an error in the derivation of this relation by Schuster and shows that the group-velocity is after all what is determined by the rotating-mirror method. He shows further that the results of Michelson's observation on carbon disulphide give a closer agreement with the refractive index with this conclusion than with the conclusion arrived at by Schuster. Lord Rayleigh suggests in his second note that by placing a lens in the path of the ray so that the fixed mirror is at its focus the rotation of the wave-front caused by the rotating mirror would be corrected on the return and we should thus find the absolute velocity V. This is evidently in error, as Schuster showed, as the rotations would be added. Even if neutralized we should not obtain V, but U. In a reply to a communication by the writer to Lord Rayleigh in 1890 as to such a method for the absolute velocity he has indicated a misapprehension on this matter and expressed his opinion as to the probable correctness of Professor Gibbs's conclusion which agrees with his first position. With the exception of Schuster the rotating-mirror method seems to be accepted by all as giving the group-velocity. The correction for the rotation between two successive wave planes which is erroneously given by Schuster would give the group-velocity and thus an agreement with the results of the rest. While the observations of Michelson on carbon disulphide give the closest agreement with this result, more exact data for specific wave-lengths are desirable in order to confirm the theoretical conclusion.

In studying the question of group and wave-velocities in connection with dispersion the following two methods occurred to the writer: The first one was for increasing the sensibility of the old methods, and the second one for observing the wave-velocity directly by means of interference. In 1889 he was invited by President Hall to Clark University to conduct, among other things, an investigation on some special problem.

The dispersion in air was selected, and a combination of the arrangements of both Fizeau and Foucault which had occurred to the writer before was adopted. The essential condition in Fizeau's method is to produce an intermittence in a beam of light. This is done mechanically by the rotation of a toothed wheel.* It is quite clear that if the wheel were fixed and the ray rotated the condition of intermittence would be fulfilled. It would then be merely a matter of arranging a suitable optical system to maintain a fixed direction for the ray while in transit between the two stations. Any such optical system would avoid the inertia inherent in a mechanical system and would thus allow of much greater speed and consequent sensibility. Through the courtesy of the Secretary of the Navy and the active assistance of Professor Newcomb, the phototachometer of the latter was secured to carry out this experiment at the University of Nebraska instead. This instrument seemed to be well suited for a preliminary experiment on account of the rectangular shape of the rotating mirror and the number of reflecting faces available. Figures 1 and 2 show the arrangement used and the path of the ray. The original apparatus was changed slightly, the two telescopes T and T' being shifted so that their axes passed through the middle of one set of faces when these stood at 45° to the same. The other additions were the lens L the grating G and the plane mirror M. The instrument was originally mounted so as to use the concave mirror belonging to the instrument itself, which had a radius of curvature of 3,000 meters. Owing to improvements about the campus it became necessary to remove the piers and discontinue the experiment for several years. It was again finally mounted in the basement of the Physical Laboratory on a much smaller scale and the flat mirror M used at a distance of about fifteen meters from the rotating mirror R. The lens L was a telescope lens of one meter focus and ten centimeters aperture. The concave mirror G had a radius of curvature of one meter and

Fig 1

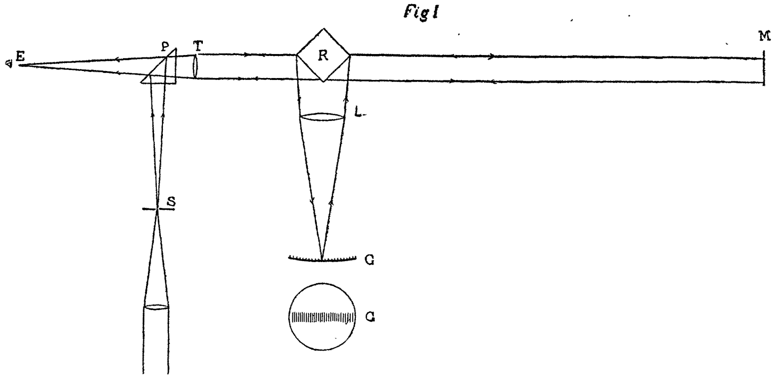
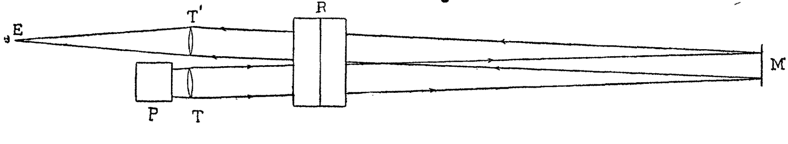


Fig 2



* In the March number of the *Philosophical Magazine* for this year, Professor Michelson describes a similar arrangement which occurred to him independently during his experiments on the motion of the ether.

an aperture of ten centimeters. A narrow strip across this mirror was divided into equal bright and dark spaces of ten to the millimeter. This was accomplished by means of a diamond making five deep strokes in each alternate space. In this way normally incident light would be returned over the same path from the bright spaces, but be scattered from the cuttings in the adjacent spaces, so that very little of it would be returned in the direction of incidence. A lens converged the sun's rays on the slit S from which a beam passed to the mirror P through the collimator lens T to the upper part of the adjoining face of the rotating mirror R from which it was reflected at 45° through one quarter of the lens L and brought to a focus on the grating mirror G. From this it was reflected through the lower opposite quarter of the lens to the lower part of the next face of R and thence reflected at 45° to the plane mirror M as parallel light. This reflected it to the upper part of the same face of R thence through L to G and back to the lower half of the face of R upon which it was first incident and thence through the observing telescope T' below the collimator, to the eye. It is clear from the diagram and from the principle that an even number of reflections of a ray system from a moving reflecting system does not alter their direction that the incident and the reflected rays from the rotating mirror R will remain parallel to each other and hence will always meet the mirror M at the same point during the rotation of R. For the moment let us assume the image of the slit S just covers one of the bright spaces. By proper adjustment of M the return image can be made to coincide with it. Usually it was displaced slightly below it so as to observe the relative positions of the two when the mirror R was rotated so as to carry the images across the grating. If now, during the time of transit from G and back, the mirror has rotated through an odd number of spaces no appreciable light will be reflected from G through T' to the eye. If the rotation corresponded to an even number of spaces, the eye would see an enfeebled image of the slit S. If the mirror were varying in speed the eye would see this image pass successively through maxima and minima, depending on the rate of change of the rotating mirror R. Suppose now the image of S covers any number of spaces on G the eye will see an image of S, but crossed by bright and dark spaces corresponding to those on G. With the corresponding variations in the speed of the rotating mirror the eye will see corresponding fluctuations in this image. In this way the eye may be able to determine the minima by comparison with the darker spaces which remain of constant intensity.

The aperture of the 'sending' telescope was 4.5 cm. but the effective aperture with the rotating mirror at 45° was only 2.5 cm. The actual spacing was .02 cm. between the bright lines, which is well within the limits of good definition. The mirror could be driven up to 250 revolutions or more per second without serious vibration. Thus the ray could be interrupted about 10,000,000 times in a second. This would give a group of waves about fifteen meters long. If the limits of resolving power and speed were used 40,000,000 interruptions could be obtained and the length of the groups could be reduced to less than four meters. Thus there would be about 6,000,000 waves in each group. As the eye can observe a change in intensity of less than one per cent., the method would thus be capable of detecting the existence of a velocity if the total distance of the mirror M were less than 2 cm. from the grating. This shows the sensibility which the combination of the methods of Fizeau and Foucault may give under very favorable conditions.

If the velocity were different for differ-

ent colors this method would be capable of showing even a very slight difference. For example the difference in velocity between the extreme red and the violet rays in carbon disulphide is about one sixteenth. For air this difference is about one part in one hundred thousand. Now an addition of five per cent. of one of these colors and a subtraction of the same amount of the other from white light will produce a perceptible change in the time. Thus if we consider a five per cent. change instead of one per cent. as mentioned previously, and multiply this by sixteen we obtain 1.6 meters as the length of a column of carbon disulphide between the grating and the fixed mirror, in which we could just detect the dispersion of light. Similarly in the case of air we have to multiply by five and by 100,000 and obtain about ten kilometers to produce the necessary dispersion in air. It is very doubtful whether this sensibility can be actually realized. In the preliminary experiments with his instrument sunlight was used. The scattered light from different parts of the system prevented the contrast between the light and dark spaces in the return image which would be necessary for such a high sensibility. By adjusting the mirror *M* so that the images of the bright spaces of the grating should fall on the dark spaces on their return, the appearance for an eclipse could be studied. In reference to the intensity of the return image, if we allow fifty per cent. less from successive reflections and remember that the angle of the grating was only one sixtieth of the circumference, we find only about one per cent. of the light available in the return image. Thus with the mechanical system of Fizeau's toothed wheel we should have one hundred times the light available for the same aperture. The preliminary experiments were made under unfavorable conditions, but indicated a greater sensibility than that heretofore obtained. With sunlight sufficient intensity would be available for the experiments just referred to. With the apparatus as mounted too much instability was experienced to obtain satisfactory definition with the rotating mirror and the motor and blower which drove it in motion. The condition of the faces of the rotating mirror also prevented distinct definition. In order to use all four faces they required refiguring and the angles between them recut. The lack of funds to make these changes and the recall of the instrument have caused the experiments to be interrupted several times and finally abandoned until the means for building apparatus more suitable for the purpose and for carrying out the experiment can be obtained. This method will give the group-velocity *U* directly according to the criticism already referred to on the method of the rotating-mirror. The possibility of obtaining important data on group-velocity in different media by so sensitive a method, and also the determination of the velocity of light in a vacuum itself warrants further efforts being made along this line. These experiments were initiated in 1889 and discontinued a half dozen years later in the way mentioned.

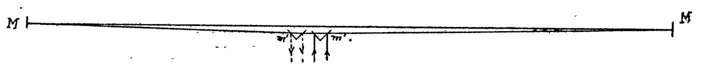
It is doubtful whether there are other methods for determining the group-velocity of greater sensibility. One method of considerable promise is by means of polarized light. Two rays of light pass through a Nichol and a half-wave plate. Each is reflected from each of two mirrors respectively. By properly focusing we should have a half-shade combination which on rotating the half-wave plate and the Nichol about the common axis would give a difference in the intensity of each return ray. Half-shade systems have a sensibility as high as one thousandth of a degree in the

limit. Such a Nichol could probably be rotated at a speed exceeding two thousand per second. Polishing machines are now run above one hundred thousand a minute. Foucault's mirror rotated eight hundred times a second. At such a speed and sensibility a velocity in light could be detected over a distance of about twenty centimeters.

Another system depending upon electric oscillations would require much the same optical system. A prism of glass between two electrodes as arranged for the Kerr 'effect' in a dielectric is placed beyond a half-wave plate and a Nichol prism which is fixed. If now the glass is subjected to electric strain by rapid oscillations, the fields from each mirror are lighted up differently and the limiting distance at which this difference is observable will be the same as stated previously. Probably oscillations several hundred millions a second would be possible with such a condenser system. Forty millions a second was the limit with a rotating mirror and grating as described above. This could probably be increased to one hundred millions a second with a suitable rotating mirror. Instead of using the Kerr 'effect' a piece of Faraday glass within a single turn of foil could be used for very high frequencies and the Faraday 'effect' employed instead. Probably the same order of sensibility could be obtained as with the former. It is hoped to make preliminary experiments on these promising methods which would probably give shorter groups or types of waves than any of the others. Here we should have not an intermittence but a property impressed at intervals upon a continuous train of waves, and the relation of the velocity of that property to the wave-velocity would have to be determined.

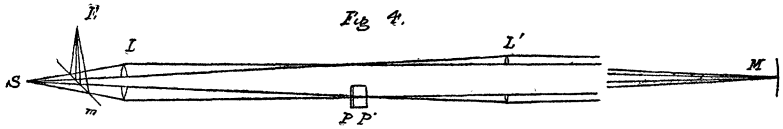
No method for determining the wave-velocity of light seems yet to have been proposed. The following arrangement occurred to the writer in 1890 while experimenting with the phototachometer as already described. Suppose that in two interfering systems of rays we could alter the length of the path of one or both of the rays during their transit from their common source to their final point of meeting, there would be a displacement of the bands depending on the relative retardation introduced into the paths during this interval. Figure 3 illustrates the first conception of the system. A beam of parallel light, from a lens, say, strikes the two adjacent faces of a rectangular mirror, each face of which reflects the rays in opposite directions to the mirrors M and M', each of which reflects the corresponding rays to the other and thence both return to the mirror, thus traversing each other's paths. If now the mirror m is displaced to the position of m' in this interval the path of one ray will exceed that of the other by twice the distance through which the mirror has moved. Knowing this distance, by means of the wave-length and the time it takes to displace the mirror a given distance, we have at once the time of displacement of the mirror from the position m to m' and thus the time of transit of any one ray around this path, and hence the wave-velocity. Considerable difficulty was experienced in devising a method of displacement of sufficiently high speed. If the mirror m is mounted on a rotating disc the rays would be reflected beyond the mirrors M and M' and the interference would be changed by

Fig 3



the angular motion of the mirror. The mechanical oscillator of Mr. Tesla suggested itself as capable of giving possibly sufficient speed. The variation in the speed which would cause the bands to be superimposed and thus obliterated rendered this method impossible. The same difficulty would be experienced with any reciprocating means. The compensation for angular movement of a disc did not seem clear and its use was abandoned for a time. Instead of this the system indicated in Figures 4 and 5 was tried. L and L' are two bisected lenses, P and P' two prisms, one of which, P', is mounted on a rotating disc so that the total thickness would be increased or diminished by its rotation. The split lens L forms two images of the beam from the slit S and one half of its aperture. In one focus the double prism is placed. The split lens L' forms coincident images on the distant mirror M, which reflects each ray back over the path through the opposite halves of the lenses to the mirror m, which reflects the light to the eye. If now the prism P' be moved in the interval of transit of this optical circuit the ray returning through it will be retarded over the other. Knowing the constants of the prism and the speed of the disc we can calculate, in this way, the wave-velocity. The interference bands in this system remained distinct during the movement of the prism over an arc of five to ten degrees. The length of this system in the preliminary experiment was about thirty meters. With sunlight and a much greater distance, distinct bands could easily be obtained with only one fiftieth the intensity, which represents the fraction of the incident light available during one revolution. Here again we are met with the difficulty of obtaining an insufficient component velocity in the direction of the ray, which is the velocity of the disc into the cosine of the angle of the prism into the index of refraction less one.

Fig. 4.



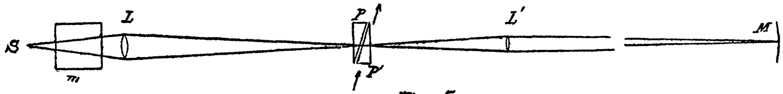


Fig. 5.

Through a fortunate idea the rotating disc was made possible and the final and most serious difficulty was overcome. Figure 6 gives the arrangement with somewhat distorted details to show the optical relations. The principle that an even number of reflections in a rotating system does not change the relation of the incident to the transmitted ray was made use of. A further difficulty had to be met in maintaining the ray upon the reflecting elements, as the rotation of the reflected ray is twice that of the reflector. If the radius of rotation is reduced one half the linear movement of any point is reduced one half. These considerations applied to two mirrors on the rotating disc met the required conditions. Thus the mirror m is placed just half the distance of the mirror m' from the center. A ray incident upon m is reflected to m' and thence reflected in some definite direction. If the disc now rotate, the ray will rotate through twice the angle, but still strike the mirror m' in the same point if

these conditions are fulfilled and will be reflected off parallel to its former direction. This relation will be maintained as long as the above conditions hold, which will be, approximately, through a considerable fraction of the circumference. This will be best satisfied when the mirror m is normal to the radius, the incident ray being thus nearly normal. The arrangement as finally adopted is shown in the figure. Light from the slit S is converged by a lens L to a half silvered plate I, one beam being reflected and the other passing through and being reflected by the mirror I'. Both converging beams strike the mirror m at the same point and are then reflected, the first beam to the adjacent face of the rectangular mirror m' and the second to the opposite face, where they form images of the slit S. The first beam is reflected to M' then to M and finally to a focus on m', while the second ray passes over the path of the first to a focus on m'. Thence the two rays trace each other's paths and are reflected and transmitted respectively by the plate I through the spectral system P to the eye, where interference bands are formed. Thus, aside from other losses by reflection, one fourth of the light reaches the eye. The distant mirror M may be concave and of the proper focus for the system; or two lenses I and I' may be used so as to obtain a greater aperture to the beam. These two

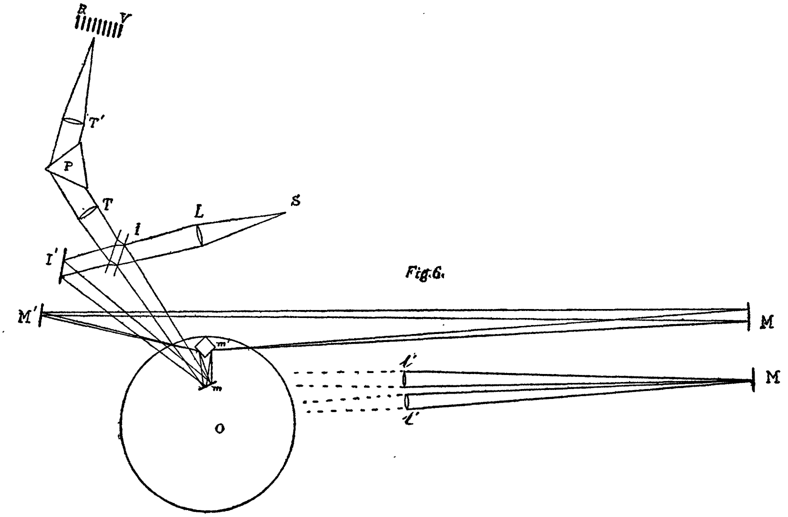


Fig. 6.

rays will in general travel over slightly different paths and hence give bands which may be conveniently analyzed by means of channeled spectra. If now the disc rotates the path of one of the rays will become greater than the other and the interference bands will shift. If a spectrum is used the bands will move across the field, increasing or decreasing in number. If the adjustment is initially made so that the paths are the same, no bands will appear until the disc is set in motion. By counting the number passing any point we can obtain the order of the interference for that wave-length, and from the dimensions and speed of the disc determine the wave-velocity for that color. From the position of the other bands at this instant we can calculate the velocity of that color. Thus we have the means at hand for obtaining the wave-velocity for all colors, from which the group-velocity for the same can at once be obtained. The radius to the disc m' is 15 cm. and a speed of 500 per second is assured. The concave mirror M has an aperture of

15 cm. and a radius of curvature of 15 M. This is the arrangement now being used. With this velocity, assuming a band can be read to one thirtieth part, a distance of only .3 cm. would show a velocity. The rays during transit may be made to pass within a tube which can be evacuated, connecting M and M'. Another arrangement may be used when M is placed at a much greater distance and is shown in the annexed diagram. I and I' are two lenses whose foci are M and their conjugate foci on each face of M' respectively.

It seems certain now that the wave-velocity in different media, as well as in vacuo, may be determined to a high degree of accuracy and that too for any color.

University of Nebraska. D. B. Brace.

PREHISTORIC PORTO RICO.*

It has been customary for the Vice-President of this Section of the Association to present in his retiring address certain general conclusions to which he has been led by his own special studies or those of his contemporaries. But it has not been regarded as out of place for him to outline new and promising fields of research or to indicate lines for future development of our science.

Late historical events have brought into our horizon new fields for conquest and opened new vistas for anthropological study. In the last years the political boundaries of the United States have been so enlarged that we have come to be regarded a 'world power,' and with this growth new colonies beyond the seas now form parts of our domain. With this new epoch certain broad scientific questions have come to present a special claim on our students, and we have been brought closer than ever before to problems concerning other races of man besides the North American Indian. Great fields of work attract our ethnologists to the far East and the islands of the Pacific, and these new problems will occupy our attention with ever-increasing interest in years to come as anthropology advances to its destined place among sister sciences. It is natural and eminently fitting that attention at this time should be directed to some of the new anthropological problems before us, and I have chosen as a subject of my address, 'Prehistoric Porto Rico,' and the Antillean race which reached its highest development in our new possession in the West Indies.

Among all the acquisitions which came to the United States by the Treaty of Paris, Porto Rico is preeminent from an anthropological point of view. Fourth in size of the Antilles, it is the most centrally placed of a chain of islands reaching from Florida to the coast of South America. Before the coming of Columbus there had developed in these islands a culture sufficiently self centered to be characteristic, and our new possession was the focus of that culture. Here was found a race living in an insular environment exceptional on the Western Hemisphere. If as the great anthrogeographers insist anthropological problems are simply geographical in their final analysis, where can we find a better opportunity to trace the intimate relationship of man's culture and his surroundings? Where was there on the American continent at the time of its discovery a people less affected by contact with other cultures or more truly the reflection of climatic conditions?

It may be truly said that important questions regarding migrations of the early inhabitants of the American continent are intimately related to the cultural character of the prehistoric race which

* Address by the Vice-President and Chairman of Section H, for 1901, at the Pittsburgh meeting of the American Association for the Advancement of Science.